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A Study on Promoting Students' Understanding of Fractions, Locations on Number Line and Linkage to Decimals Through Card Manipulation Activity

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This study aimed to explore students' problem-solving abilities in a teaching situation where card manipulation and number lines were integrated into fractions, as well as students' performance in the fraction concept test after the teaching activities. We arranged and integrated three teaching activities for students in the experimental group through story situations and classroom discussions to stimulate students' learning of fractions. The research participants were Grade 4 students in elementary school, and they were randomly divided into the experimental group and the control group. The experimental group adopted card-arranging games integrated into teaching, while the control group adopted general textbook content teaching. The experimental group activities were carried out in heterogeneous groups, and the teacher provided each group with a deck of cards and cotton thread. Through the guidance and stimulation of open-ended questions, students were allowed to discuss the results, causes, and ideas of card manipulation. We analyzed the pre-test and post-test data on the concept of fractions to compare the changes in student performance before and after the teaching. The study found that:

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(a) classifying cards helped students to concretize fractional number concepts; (b) equal division and marking of number lines strengthened the relationship between the size of marks and their spatial positions; (c) the arrangement of cards on a number line reinforced the concept of equivalence between fractions and decimals. After integrating card-arranging games into teaching, the test score of the experimental group was significantly better than that of the control group.

Keywords: number line; manipulation; fraction; card game

Background

Understanding fractions is a milestone in the development of mathematical concepts for students. Indeed, it could be viewed as a gatekeeper of future mathematical achievement (Booth & Newton, 2012). Unfortunately, many students are unable to pass through this gateway in elementary school, quickly becoming frustrated (Organisation for Economic Co-operation and Development, 2016). Saxe et al. (2013) believe that part of the difficulty for students in learning fractions lies in the problems of curriculum and teaching.

The importance of learning fraction is also emphasized in Taiwan "*Curriculum Guidelines of 12-year Basic Education: Mathematics Domain*" (Ministry of Education, 2018). It explains the learning content and sequence of fractions, including: in Grade 2, the need to perform equal division operations to understand unit fractions; in Grade 3, to combine operational activities and integer experience to learn the concept of simple fractions with the same denominator and one decimal, and to integrate the integer number line, including registration and marking, connecting the experience of number sequence and length, and understanding the meaning of comparison, addition and subtraction on the number line; in Grade 4, to expand the application of fraction learning, including understanding the meaning of equivalent fractions through operations with game plays as activities, the comparison of simple fractions with different denominators, the meaning of addition and subtraction of fractions, and the exchange of simple fractions and decimals (Ministry of Education, 2018).

Research has found that number line, like other effective representational tools, such as area or sets, can promote the effect of fraction learning (Boyer & Levine, 2015; Eriksson & Sumpter, 2021; Sidney et al., 2019). Although the benefits of number line knowledge to integers are obvious, the course materials pay limited attention to the linear model of integers and fractions, and there are no substantive suggestions for the number line's role in understanding the magnitude of fractions. To cultivate students' reasoning and problem-solving ability, we must first provide appropriate activities to promote students' participation as a means of constructing new knowledge and generating mathematical thinking. However, many studies have shown that students have weak awareness of mathematical structures (Kieran, 2022). This phenomenon just highlights the need for teachers to help students develop reasoning. If students want to have this ability, teachers must develop students' ability to reason first and provide them with preparation and support. But how to translate ideas about mathematical reasoning into usable practices that support research and design, enabling students to focus on learning specific mathematical topics? How can this focus be translated into concrete learning goals? How is student learning assessed and described in light of these instructional objectives? This is an important issue worth exploring.

In research and practice, card games are also effective in improving students' performance in mathematics knowledge. After card games are integrated into the curriculum, most students will be willing to take the initiative to learn mathematics, and even look forward to taking mathematics courses. In this study, the games activity design is based on the syllabus of the Ministry of Education (2018), and the fractional (decimal) number cards (digital representation) are designed as learning tools. Since the syllabus requires Grade 4 students to learn number lines, fractions, and decimals, they must be able to connect the experience of fractions and decimal lengths, and establish an integrated understanding of integers, fractions, and decimals through labeling and simple comparisons and calculations. This study intended to develop and strengthen this goal through numerical line teaching aids.

The specific questions of this study were:

1. Exploring the effectiveness of teaching students the concept of equivalent fractions:
   a. Exploring students' performance in placing cards on the number line;
   b. Exploring how students mark units and place cards on the number line;
   c. Exploring how students connect the equivalent concepts of decimals and fractions.
2. Examining how students perform on a fraction concept test.

Literature Review

Learning Mathematical Concepts Through Activities

The workings of the mind are an intermediate step between concrete activity and abstraction, involving specific concepts that students need to consider invoking and can help construct the correct concepts, and expectations that need to be developed. And how to go from the observation of objects to the production of concepts? Simon proposed "Learning

Through Activity" (LTA), describing mathematical concepts as: researchers' expectations or inferred expressions of student knowledge about the logical necessity involved in specific mathematical relationships (Simon & Blanton 2018; Simon et al., 2018). The acquisition of mathematical concepts and reasoning skills must go through two types of reflective abstraction:

Coordination of actions (CoA)

It consists of two parts: a goal and the actions the learner is expected to take to achieve that goal. Goals refer to the goals set by learners, and activities are the original materials for the construction of mathematical concepts. They consist of a goal and a series of actions combined to achieve the goal. The new concept involves a learning ability to predict the outcome of part or all (whole) activities without performing the action. The basic strategy of CoA is to start with a task that induces activities, the coordination of which may lead to the intended concept. Learning is not the result of seeing patterns in numbers, but is an abstraction of the learner's activities. When operating an action as part of a current activity, the action called is a component concept. The coordination that occurs can therefore be learners' more accurate thought of the coordination of concepts.

Abstraction of commonality (AoC)

It is a process of manipulating mathematical structures and classifying mathematical experiences into examples and non-examples. The key points about AoC are:

- thinking about what is "the same," which is the commonality of the objects that learners need to identify in their activities;
- this same property being the basis for identifying common mathematical structures;
- once learners able to think about common structures, there being a mathematical entity that can be named and symbolized.

The basic strategies for promoting AoC are to create tasks with informal activities that elicit students' skills to classify tasks into those that indicate activities and those that do not, and finally to be able to abstract the meaning and concepts of objects from their commonalities. Simon's perspective of LTA can lead to the following inspirations for this study (Simon & Blanton, 2018; Simon et al., 2018):

1. The formation of mathematical concepts does not happen overnight. It requires a process of reflection and abstraction; that is, there must be appropriate activities that trigger participation and motivate students to achieve their expected goals, conducive to concept formation and development.
2. To enable students to use activities to connect to common abstractions requires good task design, through the connection of sub-goals and the combination of tasks, to promote the identification of object attributes in mathematical structures.
3. Action coordination to common abstract mathematical reasoning activities allows students to observe the common attributes of operating objects to the output of symbols, helping teachers understand and analyze the output and application of students' concepts.

Fraction Learning

A robust understanding of fractions includes developing each of the fraction (including decimals) sub-constructs, and developing an understanding of a fraction as a measure, such as a length on a number line, has been identified as particularly important for students' future mathematics achievement (Sidney et al., 2019; Tian & Siegler, 2018). There is growing evidence that students' ability to think of fractions as measurable quantities affords early algebraic reasoning (Eriksson & Sumpter, 2021; Hackenberg & Lee, 2015; Kieran, 2022; Sidney et al., 2019). Saxe et al. (2013) identified from previous research the difficulties students may experience when struggling with new ideas, and discovered students' understanding of whole numbers and written notation for numbers, and their intuition for sequential relationships between linear magnitudes. Therefore, for number lines to be a resource to support understanding of fractions, knowledge of whole numbers and intuitions of order and magnitude need to be grounded in building an understanding of representations of fractions or decimals. Saxe et al. (2013) analyzed the literature and proposed core concepts crucial for students to understand number lines. The following inspirations were inspired by the design requirements of the course content:

1. providing open-ended questions can stimulate students to explore fraction learning and understand students' thinking;
2. open discussions in class can encourage students to apply the previously established definitions and principles of related number lines and guide students to solve problems;

3. the positioning and marking of the number line are the basis of fraction learning; it focuses on the operation and strategy of students' equal division, and tests the development of students' concept of fractions;
4. interactive data from discourse practices are analyzed to understand how teachers and students think about the concept of fractions.

In view of this, this study is based on the curriculum framework of Simon's LTA (Simon & Blanton, 2018; Simon et al., 2018) and Saxe et al. (2013), who designed fraction (decimal) cards for fraction learning. This study arranged three teaching activities in the form of story situations to play with classroom discussions, carried out teaching activities and used the discussion materials in the classroom to conduct linear analysis to detect students' learning performance.

Research Methods and Procedures

Participants

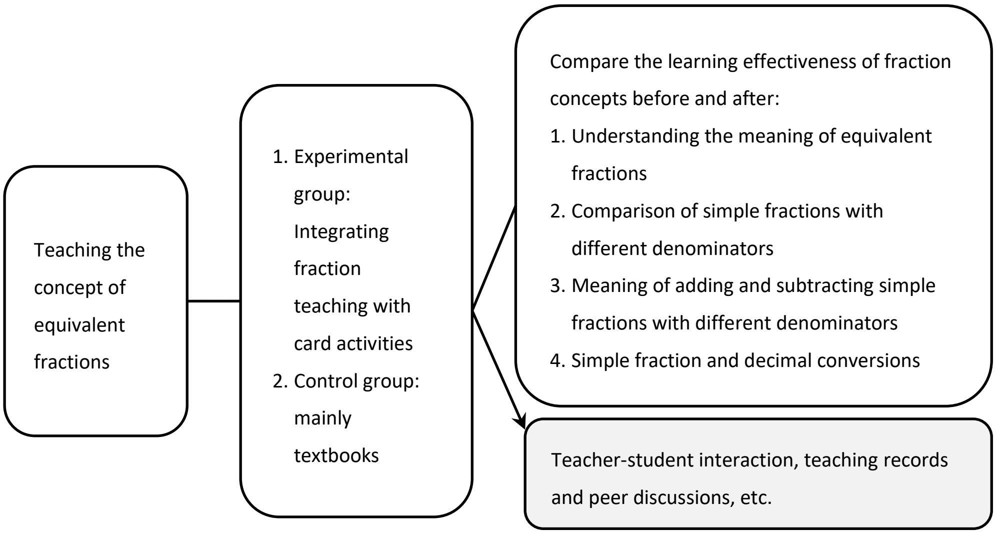
The participating school of this study is a municipal elementary school in central Taiwan. The school is located near the airport and has a substantial proportion of aboriginal students. It is currently a key aboriginal school under the Ministry of Education. Most of the parents in this school have labor jobs. Owing to their busy schedules, they have less time to spend with their children. Therefore, the ratio of students supporting students is relatively high. Their basic academic ability is weaker than that of children in urban areas, and their learning motivation is low. Teachers are very concerned about children's learning and making friends and will spend more time and thought on tutoring students. Therefore, the relationship between teachers and students is harmonious, and students will regard the school as their second home.

This study was conducted in the form of a teaching experiment. There were 21 students in the experimental group and 22 students in the control group. Most of the students in the two groups had the trait of being inattentive when studying, but they were lively, active, and good at expressing themselves. Because of this characteristic, the whole class is unable to concentrate during classroom discussions. Occasionally, the teacher asked certain questions many times, but the students did not pay attention to listening, so they raised their hands to ask the teacher afterwards; or they asked about the same question. When one question involved a lot of content that was not in line with the theme of the class, the teacher needed to remind the students many times to bring their mind back to the mathematics class. Therefore, this study intended to promote students' learning motivation through card manipulation, in addition to improving academic performance in mathematics.

Research Design and Implementation

This study adopted a quasi-experimental design. The experimental group used self-edited teaching materials for integration of teaching, and the control group used textbooks for teaching. The learning effectiveness was analyzed based on the pre-test and post-test performance of the two groups of students, and the students' multiples in the experimental group's teaching were further explored before and after the post-test. Figure 1 shows the research structure.

Figure 1: The Research Structure



Fraction Card Games Design and Regulations

With objectives being equivalent fractions, ordering numbers (fractions, decimals), and interchange of fractions and decimals, the investigator developed teaching activities using cards.

Grouping: 2 students in each group

Teaching aids: ruler (100 cm long), 52 cards as below (number is from 0 to 1)

♠ Fraction: $\frac{1}{2},\frac{2}{2},\frac{2}{5},\frac{4}{5},\frac{5}{5},\frac{4}{10},\frac{5}{10},\frac{8}{10},\frac{10}{10},\frac{40}{100},\frac{50}{100},\frac{80}{100},\frac{100}{100}$, 13 cards

♣ Decimal: 0.4 (3 cards), 0.5 (3 cards), 0.8 (3 cards), 1 (4 cards), 13 cards

♥ Fraction: $\frac{3}{2},\frac{4}{2},\frac{6}{5},\frac{8}{5},\frac{10}{5},\frac{12}{10},\frac{15}{10},\frac{16}{10},\frac{20}{10},\frac{120}{100},\frac{150}{100},\frac{160}{100},\frac{200}{100}$, 13 cards

♦ Decimal: 1.2 (3 cards), 1.5 (3 cards), 1.6 (3 cards), 2 (4 cards), 13 cards

Activity 1: Classifying cards (40 minutes)

1. Students form groups of two and put 52 cards into categories (try 5 ways). They sort cards and then write down how they classify the cards.
2. Scoring method: 1 point will be added for each correct answer.

Activity 2: Number line division and marking (40 minutes)

1. Each group discusses how to divide the cotton thread with a total length of 2 and a length of 100 into equal parts, finds the points where the cards need to be placed, and marks these points with numbers.
2. Scoring method: 1 point will be added for each correct answer, for a total of 8 points.

Activity 3: Arrangement of cards on the number line (40 minutes)

1. Players in each group are asked to find out 0, 1, and 2 on the number line after discussion, and then arrange the card values from small to large on the number line.
2. Scoring method: 8 points will be added to the group that finishes first, 7 points will be added to the group that finishes second, and so on, but no point will be given to those that do not finish.

Determine the Winner: After playing cards, the total scores are compared and the group with the highest score wins.

Administration of Fraction Teaching in Three Activities

This study arranged three teaching activities in the way of story situations and cards, with classroom discussions to stimulate students' learning of fractions. Table 1 shows the content and description of the activities.

Activity 1. The purpose is to test students' understanding of the properties of fractions and decimals, and to understand whether students have experience in learning unit fractions (decimals). *Activity 2*. According to the observation of the attributes of the card found in

Activity 2, students are required to divide the cotton thread into equal parts and mark the point numbers. They are required to use 0 as the origin at one end of the thread and 2 at the other end, establish unit fractions by equal division, carry out multi-unit and sub-unit intervals, and mark the numbers at relevant positions. *Activity 3*. Use the position of the same point on the cotton thread to test the cards on the same point. The equivalence of digital representations and the symmetry between numbers provide students with the reinforcement of equivalent fractions and the transformation of the concept of equivalent fractions and decimals.

Table 1: Description of Teaching Activities

Activities	Description of teaching activities
1. Classifying cards	• Xiaohua bought a novel card reader. This machine can identify the input cards according to the attributes of the cards given, so they can be divided into many types. If you are this smart card reader, how will these cards be classified? • Tell me what you think. • What are the characteristics of classified cards?
2. Number line division and marking	• After classifying through the card reader, Xiaohua learned that there are 52 cards in total, and they all have certain characteristics in common with each other. He wanted to gather cards with the same value to form a family. And to place these cards in the correct position from left to right in order of size, he took out a piece of cotton thread and thought about it. He marked the rope first so that these cards could find their positions smoothly later, what will Xiaohua do with this cotton thread? • Which number should be used to mark the ends of the cotton thread? • What does the denominator presented on the scorecard mean? Can you think about how to divide it equally according to the denominator? • What does the place value after the decimal point mean? How to divide the cotton thread equally according to the place value?
3. Arrangement of cards on the number line	• Xiaohua has already marked the value of this cotton thread, now you need to put the 52 cards in the proper position, how will you deal with it? Let the cards find their own home. Think about why they are members of the same family.

The implementation of these activities was to meet the requirements of the curriculum outline of the Ministry of Education (2018), including: (a) understanding the meaning of equivalent fractions — starting from natural comparison or addition and subtraction problems, let students learn from the operation; and during the discussion activities, understand the meaning and operation method of equivalent fractions; (b) comparison of simple fractions with different denominators — in the case of simple fractions with different denominators, through equivalent fractions, convert them into fractions with the same denominator and then compare fractions; (c) significance of addition and subtraction of simple fractions with different denominators — in the case of simple fractions with different denominators, through equivalent fractions, convert them into fractions with the same denominator before adding and subtracting fractions; (d) interchange between simple fractions and decimals — through equivalent fractions, when the denominators are 2, 5, 10, and 100, simple fractions and decimals are interchanged.

In order to integrate the number line and card games so that the teaching activities can be carried out smoothly, the description and questions of the situation were introduced. For example, in Activity 1: “Do you find that there are other rules for these cards? Observe the similarities and differences of cards of different suits.” Activity 2: “You are required to place the cards of decimals and fractions in the appropriate position on the number line. You can regard 0, 1, and 2 on the number line as targets, and use the distance between them as the basis for equal division based on the size of the denominator to establish unit fractions and explore the relationship between other fractions and this unit fraction, and mark its position on the number line.” Activity 3: “Are fractions and decimals interchangeable? What are the characteristics of fractions and decimals at the same position on the number line? What is the relationship between the size of the cards arranged on the number line?” The time required for teaching included four lessons, totaling 160 minutes, including pre-test and post-test. A group of 2 people would solve problems and publish together, and the tutors of the class would teach respectively.

Data Collection and Analysis

Quantitative and qualitative data were collected. Qualitative data was collected by recording the teaching process in the experimental group. Since this research required students to explore in teaching activities with card games and communicate their thinking through discussions, the data about teacher-student interaction was mainly based on qualitative analysis using triangulation. Data analyses on the three card games activities were used to answer the two research questions.

The analysis was supplemented by quantitative analysis, conducting dependent sample *t*-tests of the total scores and the scores in each part of the pre-test and post-test of a faction test. The researcher did validation to improve the reliability and validity of the research.

*Fraction test***.** Before and after the teaching activities, the researchers administered tests on concepts of fractions to compare the changes in the experimental group and the control group before and after the teaching, to compare their academic performance. The test contents included all competence indicators in the standard textbook following guidelines of curriculum standards (Ministry of Education, 2018). The question items were all multiple-choice (see Appendix). Students received 1 point for correct response and 0 otherwise. Four items were allocated for each indicator, resulting in a four-part test with the highest possible score of 16 points. The test content was reviewed and revised by experts in mathematics education. Table 2 shows the distribution of question items.

Table 2: Distribution of Question Items in Equivalent Fraction Concept Test

Equivalent fraction concept test	Mathematical concept	Number of questions	Total score
Part 1	Understanding the meaning of equivalent fractions	4	4
Part 2	Comparison of simple fractions with different denominators	4	4
Part 3	Meaning of adding and subtracting simple fractions with different denominators	4	4
Part 4	Simple fraction and decimal conversions	4	4

Pre-test was carried out before the teaching, and then the instructor conducted the teaching. The teaching time was 160 minutes. After the teaching, post-test was carried out after one week (30 minutes each for the pre-test and post-test).

Findings and Discussion

This section reports the findings of data analysis from each teaching activities based on video recordings of lessons.

Classifying Cards Help Students to Concretize Fractional Number Concepts

T: Look at the cards placed on the number line and sort them into groups. Explain what characteristics did you use to sort them.

S1: I divided the cards by color: red and black. There are 26 cards in each group.

S2: There are four suits: spades, clubs, hearts, and diamonds. There are 13 cards in each group and 52 cards in total.

S3: There are 26 cards with decimals and integers and 26 cards with fractions.

S4: Of the 52 cards, there are 26 cards with numbers that are less than or equal to 1. The other 26 cards have numbers that are greater than 1 and less than or equal to 2.

Classifying cards displayed images of the students sorting the cards by characteristics such as color, suit, integers, fractions, and decimals. Next, the teacher asked the students to sort the cards based on their values:

T: Everyone, how do the spades and the hearts differ?

S2: Both types of cards have fractions on them, and the denominators are all 2, 5, 10, or 100.

S3: The values of the fractions on the spade cards are all less than 1 (proper fractions) or equal to 1 ($\frac{2}{2}$, $\frac{5}{5}$, $\frac{10}{10}$, $\frac{100}{100}$), and the values of the fractions on the hearts cards are all greater than 1 (improper fractions) or equal to 2 ($\frac{4}{2}$, $\frac{10}{5}$, $\frac{20}{10}$, $\frac{200}{100}$).

T: What is the relationship between the clubs and the diamond cards?

S1: Both types of cards have decimals on them.

S5: The values of the fractions on the club cards are all less than 1 (pure decimals) or equal to 1, and the values of the fractions on the diamond cards are all greater than 1 (mixed decimals) or equal to 2.

T: How are the spades and the clubs the same or different?

S1: They are all black cards.

S3: Some of them are equal, like $\frac{40}{100}$, $\frac{50}{100}$, $\frac{80}{100}$, and $\frac{100}{100}$ equal 0.4, 0.5, 0.8, and 1.

T: How are the hearts and the diamonds the same or different?

S1: They are all red cards.

S3: Some of them are equal, like $\frac{120}{100}$, $\frac{150}{100}$, $\frac{160}{100}$, and $\frac{200}{100}$ equal 1.2, 1.5, 1.6, and 2.

Through the teacher-student interaction, we understand that: students classified the cards of decimals and fractions through games; learned the terms of improper fractions, proper fractions, decimals, and pure decimals; and found out the corresponding values of fractions and decimals, with the concept of fractions and the ability to classify fractions and decimals. It classified the observed fractional denominators and decimal place values, and further provided the basis for equal division and marking of seat number lines.

Equal Division and Marking of Number Lines Strengthen the Relationship Between the Size of Marks and Their Spatial Positions

Previous research has shown that the ability to place decimals and fractions in appropriate positions on the number line is where students are weak (Eriksson & Sumpter, 2021; Saxe et al., 2013; Sidney et al., 2019). In this study, following instructions from the teacher, most of the students could regard 0, 1, and 2 on the number line as subjects, use them to divide the number line into equal parts depending on the denominator, show the unit fractions, explore the relationships between these unit fractions and other fractions, and mark their locations on the number line. The teacher-student discussions are as follows:

T: Please think about it, if you want to put the cards on the number line in order from small to large, where would you put them? (We can set 0 at the beginning of the left end of the number line)? We will help them set the position on the cotton line now, how to find out the position of 1 and 2 on the number line?

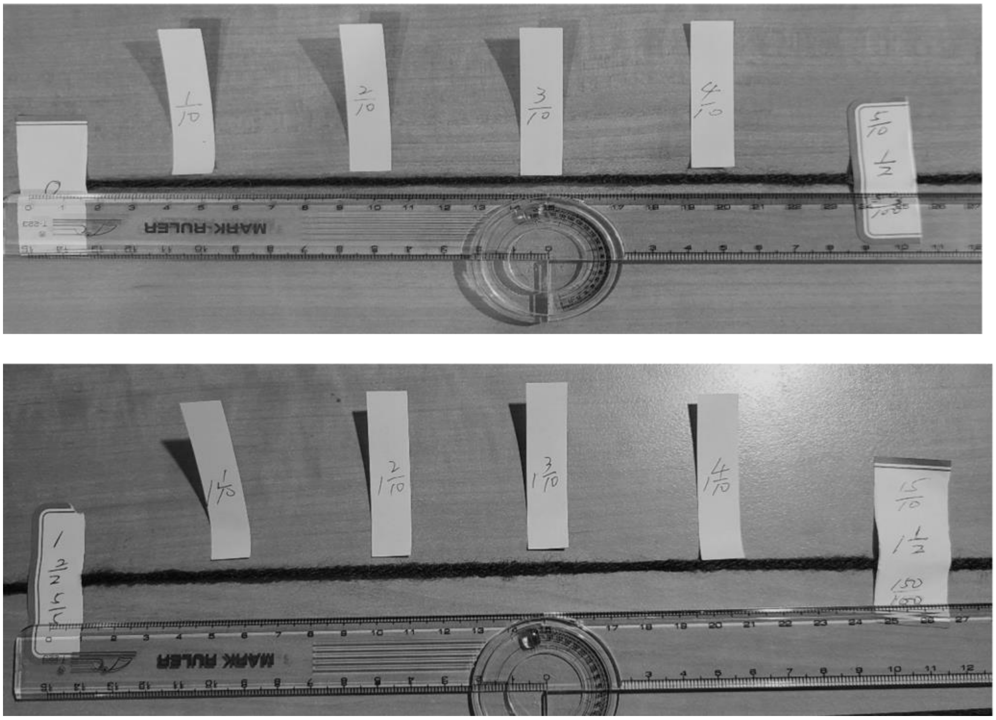
S3: We regarded the left end of the cotton thread as 0, and the other end as 2, and the numbers from left to right were larger. First, we folded the cotton thread in half, the middle point was 1, then folded the 0 and 1 cotton thread in half from left to right, the half-folded place was $\frac{1}{2}$, and the 1 and 2 cotton thread on the right was folded in half from right to left After that, its distance was also $\frac{1}{2}$, but to the right of 1, so it should be marked as $1\frac{1}{2}$ (Figure 2). Then we wanted to fold the cotton thread of 0 and 1 into 5 sections, but we couldn't fold it out, which was a bit difficult.

T: S3, why do you fold the cotton thread between 0 and 1 into 5 sections?

S3: Because there are $\frac{2}{5}$ and $\frac{4}{5}$ cards with a denominator of 5 in the cards, it is necessary to find $\frac{1}{5}$ first before finding the positions of other fractions with a denominator of 5.

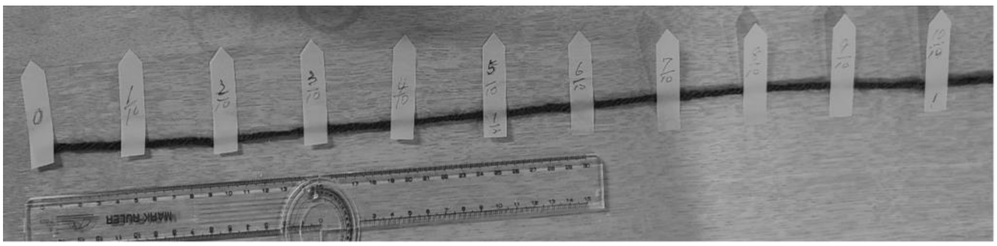
T: But there are fractions with denominators of 10 and 100 in the cards! Does that mean you need to fold the cotton thread into 100 sections? Think again about how you want to divide the cotton thread into equal parts.

Figure 2: S3's Representation of Number Line Marking and Equal Division



S4: This cotton thread is 100 cm, and it is 50 cm after being folded in half. We used a ruler to divide the 50 cm cotton thread into 10 sections, each section is 5 cm, which represents $\frac{1}{10}$, and marks $\frac{1}{10}$ (0.1) on the 5 cm, the top of 10 cm is marked as $\frac{2}{10}$ (0.2), and the top of 15 cm is marked $\frac{3}{10}$ (0.3) (Figure 3).

Figure 3: S4's Representation of Number Line Marking and Equal Division



T: Why does 5 cm represent $\frac{1}{10}$ instead of 10 cm representing $\frac{1}{10}$?

S4: We originally thought that 10 centimeters were $\frac{1}{10}$, but we found that 10 $\frac{1}{10}$s are 1, but the largest number of these cards is 2, and these cards can only be put on the cotton thread as 2, so there should be 10 segments between 0 and 1 and 10 segments between 1 and 2. The total length has 20 segments, 100 ÷ 20 = 5, and each segment is 5 centimeters.

T: Why is the full length divided into 20 segments? Can you explain more clearly?

S4: 5 cm represents $\frac{1}{10}$, 10 cm represents $\frac{2}{10}$, 50 cm represents $\frac{10}{10}$, which is equal to 1; 55 cm represents $1\frac{1}{10}$, 60 cm represents $1\frac{2}{10}$ … 100 cm represents $\frac{20}{10}$, equal to 2.

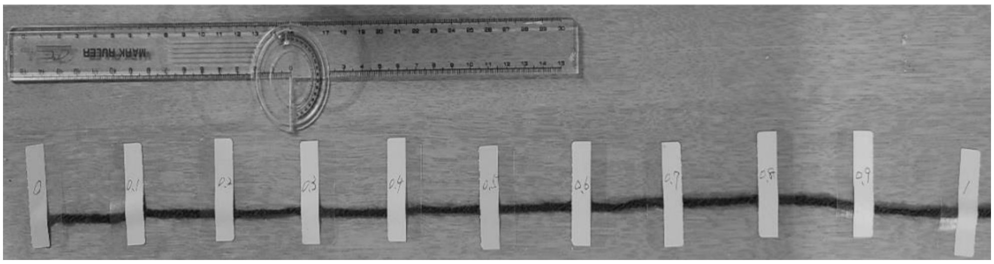
T: But you didn't deal with fractions with a denominator of 100!

S4: Yes! The values of $\frac{120}{100}$、$\frac{150}{100}$、$\frac{160}{100}$、$\frac{200}{100}$ are 1.2, 1.5, 1.6, and 2 respectively, which are also $\frac{12}{10}$、$\frac{15}{10}$、$\frac{16}{10}$、$\frac{20}{10}$. So it can be divided into 10 segments.

S3: After we listened to the explanation of S4, we also used 5 cm as a segment to divide the cotton thread. At the $\frac{1}{2}$ point, there are 5 segments of 25 cm, and $1\frac{1}{2}$ is 75 cm, and there are 15 segments.

S5: After we folded the cotton thread in half, we divided 5 cm into 1 section, representing 0.1, then 1 is 10 (0.1), that is, we needed to divide 1 into 10 sections. We used 5 cm as a benchmark, and took 5 cm as the length folding down, you could get 0.2, 0.3, 0.4 … (Figure 4).

Figure 4: S5's Representation of Number Line Marking and Equal Division



S1: We used a ruler to measure the length of this number line to be 100 cm, so I divided it into 2 equal segments, marking 1 at 50 cm, $\frac{1}{2}$ at 25 cm, and $1\frac{1}{2}$ at 75 cm, and 2 at 100 cm, and then 5 cm was divided into 1 section as $\frac{1}{10}$, 2 sections as $\frac{2}{10}$, 3 sections as $\frac{3}{10}$, 4 sections as $\frac{4}{10}$.

At the beginning of the number line equalization and marking activities, it can be seen that students were confused about the problem that the overall amount of cotton thread is 2. As Saxe et al. (2013) and Steffe & Olive (2010) have found, many students find it difficult to understand linear distance. The measurement attribute is used for fractions, but after peers and public discussions, you can find the position of the unit fraction through the guidance of measurement and equal division operations, understand the part of the denominator in the fraction that represents the whole, and use 5 cm as the benchmark for the unit fraction (decimal), iterating on the cotton thread to complete the marking of the number thread value. During the process of marking, students discover that the number line has the characteristics of unit fraction iteration and the concept of space symmetry from the action of folding in half; that is, they understand the meaning of the equivalent value of each unit interval, and learn the definition and principles of the number line. Constructing and applying when solving number line problems meets the requirements of Saxe et al. (2013) for using number lines to learn fraction concepts.

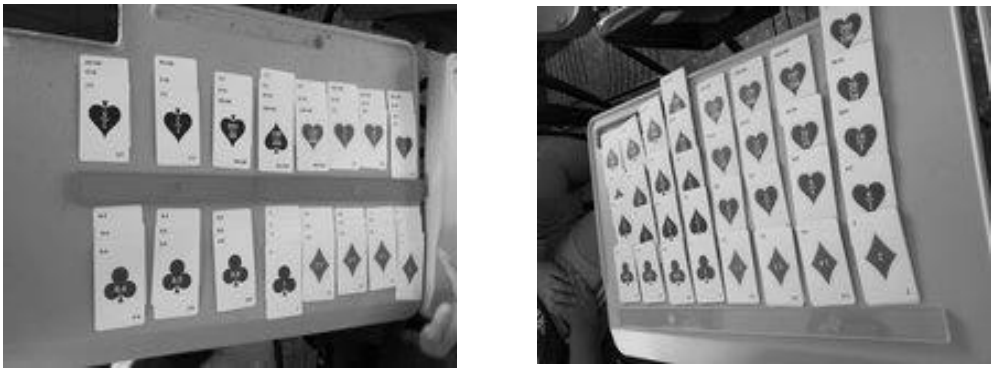
The Arrangement of Cards on the Number Line Strengthens Linkage of Fractions to Decimals and the Concept of Equivalence

Teachers can help students use the concept of the same position of fractions and decimals on the number line, guide students to understand equivalent fractions, fraction extension and reduction strategies, and the conversion of fractions and decimals, as the basis for future addition and subtraction of fractions with different denominators. The description of teacher-student interaction in the classroom is as follows:

T: Place the four suits (spades, clubs, hearts, and diamonds) on the number line. Where would you put them? (Is the left end of the ruler set as 0?) How can you find the locations of 1 and 2 on the ruler?

S3: I would use a ruler to measure the length of this number line, which is 100 cm, so I marked the 50-cm mark as 1, the 25-cm mark as $\frac{1}{2}$, the 12.5-cm mark as $\frac{1}{4}$, and the 100-cm mark as 2. Then I placed the cards at different marks depending on the numbers on them (Figure.5).

Figure 5: Images of Students Placing Cards on Number Lines



T: What have you discovered after placing these cards on the number lines?

S6: When the value is the same. I put them together.

T: How did you know they were the same?

S6: I could easily locate them if they are equal to 1 or 2, for the others that need checking with equivalence I needed to spend more time.

T: How do you know this card ($\frac{20}{10}$) is the same as 2?

S6: Since $\frac{10}{10}$ means 1, the numerator of $\frac{20}{10}$ is twice the denominator.

Through the position of the card on the number line, after identification and comparison, students understand that the fractional (decimal) numbers placed at the same point on the number line at the same time have the same value, which can be called equivalent fractions. Regarding the decimal card, these fractions can just be converted into decimal values that are easier for students to understand, and it is easier to solve the subsequent interchange problems of fractions and decimals.

The activity design of the number line with card games in this study is based on the ability indicators of the Ministry of Education's (2018) curriculum outline. Using the card games as a scaffold, guiding students to try to divide and mark the number line and place the cards on the appropriate position of the number line, through the operation of the games and visual comparison, the students learned that the same point on the number line can represent many fractions and decimals with different denominators at the same time, and through the operation of equal division of the distance between the logarithmic line segment and the space, the understanding of unit fractions was enhanced, as a basis to iterate to find the position of other score cards, to understand the relationship between cards, and to use the position placed on the number line to correctly compare the size of scores, understand the meaning of equivalent scores, and strengthen the ability to convert fractions and decimal symbols (Eriksson & Sumpter, 2021; Hackenberg & Lee, 2015; Sidney et al., 2019). This result is in line with the research results of Saxe et al. (2013), indicating that integrating number lines and card games into fractions courses can improve students' understanding of the concept of fractions, because during the course of teaching activities, students can extract what they previously learned. Students learned that the ability to equalize, the ability to understand unit fractions and the relationship between fractions and decimals through the arrangement of the number line space position; they also learned that fractions are a quantity and are composed of numerators and denominators, and concepts of fractions related to mathematical topics.

Improvement in Fraction Test Scores

Table 3 shows the performance of students in the experimental group and the control group on the fraction concept test before the teaching. The average score of the experimental group in the pre-test was 12.75 points and the standard deviation was 1.28 points; the average score of the control group in the pre-test was 12.20 points and the standard deviation was 1.67 points. In terms of the overall *t*-test, as shown in Table 3, t =.145 (p >.001), not reaching the significant level, but the variation among students in the control group was larger. There was no significant difference in the pre-test scores, which means that the overall learning starting points of the two groups before the teaching were equal; that is, the starting points of the students in the experimental group and the control group were equal.

Table 3: Independent Samples *t*-test of Overall Performance in the Pre-test

Group	*n*	*M*	*SD*	*t* value	*p*
Experimental	21	12.75	1.28		
Control	22	12.20	1.67	.145	.886

Table 4 shows the performance of the experimental group and the control group after the teaching. The average score of the experimental group in the post-test was 14.83 points and the standard deviation was 0.90 points; the average score of the control group in the post-test was 12.54 points and the standard deviation was 1.27 points. In terms of the overall *t*-test, as shown in Table 4, t = 3.50 (p <.001), reaching the significant level; that is, there was significant difference in the post-test scores of the experimental group and the control group in the test. It means that after the experiment, the overall learning performance of the experimental group was better than that of the control group. That is, after the "Cards Manipulation Integrating Fractions" teaching, the overall learning performance of the students in the equivalent fractions unit was improved.

Table 4: Independent Samples *t*-test of Overall Performance in Post-test

Group	*n*	*M*	*SD*	*t* value	*p*
Experimental	21	14.83	0.90		
Control	22	12.54	1.27	3.50	.000***

*** p < .001

Conclusion and Recommendation

The purpose of this study was: (a) exploring the effectiveness of teaching students the concept of equivalent fractions, including students' performance in placing cards on the number line, students marking units and placing cards on the number line, and students connecting the equivalent concepts of decimals and fractions; (b) examining how students perform on a fraction concept test. The study found that the classification of cards, number line equalization and marking, and the ordering of card positions on the number line, can help students understand and compare fractions (Eriksson & Sumpter, 2021; Saxe et al., 2013; Sidney et al., 2019; Tian & Siegler, 2018), and strengthen the learning of decimals. The conversion between fractions and fractions establishes the concept of equivalence. After the teaching, the test performance of the experimental group was significantly better than that of the control group.

Another purpose of this study was to determine whether the number line with card manipulation are effective tools for teaching fractions. The card classification, number line division and marking, or arrangement of cards on number lines were found to help students understand fractions. Cards with fractions or decimals presented many strategies for solving problems, and through the equivalence of fractions and decimals, the conversion between fractions and decimals was strengthened, and the arithmetic ability of addition and subtraction of fractions was also promoted (Eriksson & Sumpter, 2021; Hackenberg & Lee, 2015; Sidney et al., 2019).

The use of card manipulation provided good opportunities for fractional learning and problem-solving. The appropriate learning method encourages students to participate. The fraction cards used in this study can be used as a lesson in addition to formal lessons. In particular, high quality-designed activities promote the achievement of students' learning goals, which strengthens the importance of LTA proposed by Simon & Blanton (2018); that is, good activity design connections can promote the abstraction of common concepts. However, there were still individual differences in the students' problem-solving performance. In the course of the activity, situations such as the exclusion of operation participation or low motivation for discussion were found. It is recommended that when incorporating card manipulation into the curriculum, teachers must also consider the problems caused by students' cognitive changes in the learning environment. Finally, there were also limitations to this research tool and its application is limited. Students tended to use decimals and avoid using the addition of two fractions with different denominators. In addition, the combination of spades and clubs' cards presented relatively small numerical values. In the future, they can be combined with other cards with larger values to perform fractional multiplication and division operations with larger values.

It is noted here that the concept of fractions involves complex knowledge, abilities, and prior experiences, such as equivalent fractions, unit fractions, fraction comparisons, and decimals. Although in card manipulation, teachers should allow students to participate and explore freely, students, however, might easily lose their direction and fail to grasp the learning goals. Therefore, with the pressure of limited time, teachers can provide instructions on how to manipulate, hoping that through the design and questioning of questions, students will be able to follow the steps and have classroom discussions. This can assist students in completing assignments and building the concept of fractions.

References

Booth, J. L., & Newton, K. J. (2012). Fractions: Could they really be the gatekeeper's doorman? *Contemporary Educational Psychology*, *37*(4), 247–253. https://doi.org/10.1016/j.cedpsych.2012.07.001

Boyer, T. W., & Levine, S. C. (2015). Prompting children to reason proportionally: Processing discrete units as continuous amounts. *Developmental Psychology*, *51*(5), 615–620. https://doi.org/10.1037/a0039010

Eriksson, H., & Sumpter, L. (2021). Algebraic and fractional thinking in collective mathematical reasoning. *Educational Studies in Mathematics*, *108*(3), 473–491. https://doi.org/10.1007/s10649-021-10044-1

Hackenberg, A. J., & Lee, M. Y. (2015). Relationships between students' fractional knowledge and equation writing. *Journal for Research in Mathematics Education*, *46*(2), 196–243.

Kieran, C. (2022). The multi-dimensionality of early algebraic thinking: Background, overarching dimensions, and new directions. *ZDM — Mathematics Education*, *54*(6), 1131–1150. https://doi.org/10.1007/s11858-022-01435-6

Ministry of Education. (2018). 十二年國民基本教育課程綱要：國民中小學暨普通型高級中等學校——數學領域 [*Curriculum guidelines of 12-year basic education: Mathematics domain*]. https://www.naer.edu.tw/upload/1/16/doc/1331/十二年國民基本教育課程綱要國民中小學暨普通型高級中等學校-數學領域.pdf

Organisation for Economic Co-operation and Development. (2016). *PISA 2015 results (Volume I): Excellence and equity in education*. https://doi.org/10.1787/9789264266490-en

Saxe, G. B., Diakow, R., & Gearhart, M. (2013). Towards curricular coherence in integers and fractions: A study of the efficacy of a lesson sequence that uses the number line as the principal representational context. *ZDM — Mathematics Education*, *45*(3), 343–364. https://doi.org/10.1007/s11858-012-0466-2

Sidney, P. G., Thompson, C. A., & Rivera, F. D. (2019). Number lines, but not area models, support children's accuracy and conceptual models of fraction division. *Contemporary Educational Psychology*, *58*, 288–298. https://doi.org/10.1016/j.cedpsych.2019.03.011

Simon, M. A., & Blanton, M. (2018). Learning through activity: Studying and promoting reflective abstraction of mathematical concepts (Special section). *The Journal of Mathematical Behavior*, *52*, 92–229.

Simon, M. A., Kara, M., Placa, N., & Avitzur, A. (2018). Towards an integrated theory of mathematics conceptual learning and instructional design: The Learning Through Activity theoretical framework. *The Journal of Mathematical Behavior*, *52*, 95–112.

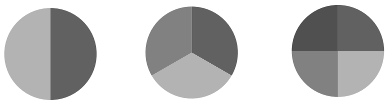
Steffe, L. P., & Olive, J. (2010). *Children's fractional knowledge*. Springer.

Tian, J., & Siegler, R. S. (2018). Which type of rational numbers should students learn first? *Educational Psychology Review*, *30*(2), 351–372. https://doi.org/10.1007/s10648-017-9417-3

Appendix: Equivalent Fraction Test (Examples)

Equivalent Fractions

4. (   ) There are many ways to make a circle. The three figures below are the circles that Hsiao-Mei, Hsiao-Ming, and Hsiao-Hua made using slices equaling $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ circles. Each of the three children described what they did. Please choose the correct description(s).



Hsiao-Mei: I made a circle with two $\frac{1}{2}$, which equals $\frac{2}{2}$, and that is 1. Hsiao-Ming made a circle with three $\frac{1}{3}$, which equals $\frac{3}{3}$, and that is also 1. Hsiao-Hua made a circle with four $\frac{1}{3}$, which equals $\frac{4}{4}$, and that is also 1. So $1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4}$.

Hsiao-Ming: I used six $\frac{1}{3}$ to make two circles, which is $\frac{3}{6}$. Hsiao-Hua used eight $\frac{1}{4}$ to make two circles, which is $\frac{8}{4}$. Hsiao-Hua used more pieces, so $\frac{8}{4}$ is greater.

Hsiao-Hua: No, no, look. I used eight $\frac{1}{4}$ to make two circles, which is $\frac{8}{4}$. Hsiao-Ming used six $\frac{1}{3}$ to make two circles, which is $\frac{6}{3}$. But we both made two circles, so $\frac{6}{3} = \frac{8}{4} = 2$.

A. Only Hsiao-Ming is right.
B. Hsiao-Mei and Hsiao-Ming are right.
C. Hsiao-Mei and Hsiao-Hua are right.
D. All of them are wrong.

Comparison of Simple Fractions With Different Denominators

1. ( ) One bag of candy contains 24 candies. Da-Yung has $\frac{7}{12}$ bags, Hsiao-Hu has $\frac{3}{4}$ bags, and Jiang-Jiang has $\frac{2}{3}$ bags. Which of the following statements is correct?

   A. Da-Yung's candy > Hsiao-Hu's candy > Jiang-Jiang's candy
   B. Hsiao-Hu's candy > Da-Yung's candy > Jiang-Jiang's candy
   C. Hsiao-Hu's candy > Jiang-Jiang's candy > Da-Yung's candy
   D. Jiang-Jiang's candy > Hsiao-Hu's candy > Da-Yung's candy

Adding and Subtracting Fractions With Different Denominators

3. ( ) To eat healthily, Mom decided to start cooking meals at home. She went to the supermarket and bought $\frac{3}{10}$ kilograms of rice, but she felt that it wasn't enough, so she had Dad pick up another $\frac{1}{2}$ kilograms from the supermarket on his way home from work. How many kilograms of rice is there at home right now?

   A. $\frac{8}{10}$ kilograms

   B. $\frac{4}{12}$ kilograms

   C. $\frac{2}{8}$ kilograms

   D. $\frac{2}{10}$ kilograms

Simple Fraction and Decimal Conversions

2. (   ) It is $\frac{672}{100}$ kilometers from home to my uncle's house. Which of the following decimals is this equivalent to?

   A. 0.672 kilometers
   B. 6.72 kilometers
   C. 100.672 kilometers
   D. 672.100 kilometers

透過紙牌操作活動促進學生對分數、數線位置與小數連結的理解

陳嘉皇、梁淑坤

摘 要

本研究目的在於探究學生在紙牌操作和數線融入分數的教學情境中解決問題的能力，以及學生在教學活動結束後在分數概念測驗中的表現。我們對實驗組學生透過故事情境和課堂討論的方式安排融入了三項教學活動，以激發學生對分數的學習。研究樣本為小學四年級學生，隨機分為實驗組和對照組，實驗組採取排卡操作融入教學的方式進行，控制組則採取一般教科書內容教學。實驗組活動以異質分組方式進行，老師為每個小組提供一副紙牌和棉線。透過開放式問題的引導和刺激，使學生討論紙牌操作的結果、原因和想法。本研究對分數概念進行了前、後測資料的分析，以比較教學前、後學生表現的變化。主要發現有：（1）分類牌卡幫助學生具體化分數概念；（2）數線的等分與標記強化了學生分數大小與其空間位置的關係；（3）數線上牌卡的排列強化了分數與小數之間的等價概念。經融入教學後，實驗組分數測驗明顯優於控制組學生的表現。

關鍵詞：數線；操作；分數；紙牌遊戲

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