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Mathematics Anxiety and Learning Strategy as Critical Factors Affecting Geometric Learning of Primary School Students

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This research examined how the core concepts of geometry, learning strategies and mathematics anxiety predict geometry learning achievements, and whether various variables can explain students' learning achievements at different levels of geometric thinking. A total of 108 children (aged 10–11) completed the tests of van Hiele geometric thinking levels, prior knowledge in geometry, and geometry learning achievement, as well as assessments of mathematics learning strategies, core concepts of geometry, and mathematics anxiety. Of them, 78 children were identified as Level 1 (n = 40) or Level 2 learners (n = 38) according to the van Hiele geometric thinking levels. Eight lessons of this study were conducted, and the content of each lesson was based on a unit of geometry. Results indicated that mathematics anxiety and mathematics learning strategy explained the geometry learning achievement of Level 1 and Level 2 learners respectively. These results highlight the different factors affecting geometry learning achievement of Level 1 and Level 2 learners and the importance of mathematics anxiety and mathematics learning strategy to learners' performance in geometry learning.

Keywords: core concepts of geometry; learning achievement; learning strategy; mathematics anxiety; van Hiele geometric thinking levels

Introduction

Mathematics is a powerful learning tool; students can use mathematics to extend and

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apply their knowledge in other curriculum areas (Rifat, 2007). However, there are different factors affecting mathematics achievement. Byrnes and Wasik (2009) examined factors that influenced the learning achievement of students in lower grades in mathematics and found that students' scores were influenced by their pre-existing mathematics skills. Verdine et al. (2014) pointed out that prior knowledge of mathematics can accurately predict a student's subsequent mathematics achievement. Besides prior knowledge, Soufi et al. (2014) argued that learning strategies were a strong predictor of students' learning achievement. Núñez-Peña et al. (2013) found that students' mathematics anxiety was also closely related to their learning achievement in mathematics. Ramirez et al. (2016) also indicated that the mathematics anxiety of students in lower grades also resulted in lower learning achievement in mathematics. However, elementary school students' mathematics anxiety is often overlooked during teaching (Jameson, 2013).

Geometry is a core component of mathematics and involves understanding fundamental concepts of geometric shapes, including dots, lines, parallel, figures, congruence, and symmetry (Dehaene et al., 2006). Geometric thinking was divided by van Hiele into five levels: visualization, analysis, abstraction, deduction, and rigor (Abdullah & Zakaria, 2013). van Hiele (1986) indicated that the five levels were arranged in sequence, and learners must have the concepts and strategies of the previous level to learn in the next level effectively; most primary school students are in Level 1 to Level 3.

Current studies have already demonstrated the influence of prior knowledge, mathematics anxiety, and learning strategies on students' learning achievement in mathematics. However, the effects of these variables on the geometrical performance of students with different levels of geometric thinking require further research. This study ascertains the correlation between mathematics anxiety, learning strategies, core concepts, prior knowledge, and students' learning achievement in geometry. It analyzes the effects of mathematics anxiety, learning strategies, core concepts, and prior knowledge on students' learning achievement with different levels of geometric thinking.

Literature Review

Core concepts of Geometry

Geometry is the deepest and oldest human reasoning product; it is an important discipline of mathematics and has been acknowledged as a domain of mathematics that has the capacity to enliven mathematics (Chambers, 2008; Naidoo & Kapofu, 2020). However, its foundation remains elusive and needs further exploration (Dehaene et al., 2006), such as the core concepts of topology (e.g., connectedness), Euclidean geometry (e.g., line, point, parallelism, and right angle), basic geometrical figures (e.g., square, triangle, and circle), symmetry figures, metric properties (e.g., equidistance of points) and geometric transformations (Dehaene et al., 2006). Geometry is important and inseparable from daily life (W. Y. Hwang et al., 2020; Post et al., 1991). However, it was found that students often failed to understand geometry concepts and solving geometric problems due to a lack of experience of applying geometric concepts in daily life (W. Y. Hwang et al., 2020). Umam and Kowiyah (2018) indicated that primary students faced many difficulties in solving a geometry problem, including perimeter and area; for example, students who believed that a shape with a larger area must have a larger perimeter while shape with the same perimeter must have the same area may fail in solving unusual geometry problems (Skagerlund et al. 2019).

Dehaene et al. (2006) designed a test to probe the Munduruku's intuitive comprehension of the basic concepts of geometry, including points, lines, parallelism, figures, congruence, and symmetry. They designed an array of six images, five of which instantiated the desired concept while the remaining one violated it. There are many ways in which the participants could have selected an odd picture out of the six, including size, orientation, or personal preference. This research uses the test for the core concepts of geometry.

Prior Knowledge

Hailikari et al. (2008) indicated that almost all educational studies had acknowledged the significant influence of prior knowledge on learning (e.g., Ausubel, 2000; Dochy, de Rijdt, et al., 2002; Thompson & Zamboanga, 2004). The knowledge that the learner already possesses in a particular subject influences the acquisition of new knowledge (Hailikari et al., 2008; Dochy, de Rijdt, et al., 2002). Therefore, prior knowledge generally serves as a reliable predictor of student achievement. The influence of prior knowledge has been explored in many different academic content fields, and 95% of the studies report positive and facilitative effects of prior knowledge on learning (Hailikari et al., 2008; Dochy, Segers, et al., 1999). Research has also shown that domain-specific prior knowledge in particular influences student achievement (Hailikari et al., 2008; Dochy, 1992). Domain-specific knowledge is one's knowledge of a specific content area, such as mathematics (Hailikari et al., 2008; Dochy, 1992).

Mathematics Learning Strategy

Yip (2013) indicated that learning strategies influenced students' academic performance and that there was a significant difference in the manner in which students with high and low academic achievement applied learning strategies. Selçuk et al. (2011) conducted a teaching experiment that taught students in the experiment group learning strategies such as questioning, drawing conclusions, and graphics organization; they found that students who were taught learning strategies had significantly better learning achievement than students who were not. Wang et al. (1994) also noted that learning strategies can effectively improve learning performance. Many studies have shown that students who are capable of using learning strategies improve their learning achievement and allow them to become more experienced in learning (K. L. Lau & Chan, 2001; Valentine et al., 2004; Vermunt & Vermetten, 2004). Cleary and Chen (2009) noted that students with higher learning achievement in mathematics were more capable of monitoring the learning strategies they used compared with other students. Chianson et al. (2010) indicated that students could retain circle geometry concepts, subject to the cooperative learning strategy. Generative learning strategy effectively improves students' performance in geometry (Bot, 2018). The search for new strategies, techniques, or methods will help students overcome their learning mathematics challenges (Bot, 2018). However, no related study indicated how the learning strategy affects students at different levels of geometric thinking.

Hopfenbeck and Maul (2011) indicated that the assessment of academic achievement in the fields of science, reading, and mathematics — the Programme for International Student Assessment by the Organisation for Economic Co-operation and Development — employed self-report, Likert-type surveys to measure a variety of non-achievement variables, such as those related to interest in academic subjects, motivation, self-concept, regard for the environment, and learning strategies. However, there is a paucity of evidence demonstrating that students understand and interpret the items on these scales and provide responses to them in a manner consistent with researchers' expectations. Therefore, this study uses the scale developed by Hopfenbeck and Maul to examine the relationship between learning strategies and learning achievement.

Mathematics Anxiety

Mkhize (2019) indicated that Richardson and Suinn (1972) defined mathematics anxiety as "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations." Núñez-Peña et al. (2013) found that students' mathematics anxiety was closely related to their learning achievement in mathematics. Wahid et al. (2014) indicated that students' mathematics performance was, to a great extent, determined by their mathematics anxiety, meaning that high mathematics anxiety will result in low mathematics grades. Ramirez et al. (2016) also found that the mathematics anxiety of students in lower grades also resulted in lower learning achievement in mathematics. García-Santillán et al. (2016) further indicated that anxiety influenced student's performance through the learning process. Suinn et al. (1988) found that mathematics anxiety scores were negatively and inversely correlated with performance on mathematics in their study of elementary school children in grades 4 through 6 (Suinn et al., 1988; S. S. Wu et al., 2012). Skagerlund et al. (2019) indicated that mathematics anxiety persisted into adulthood because of avoidance behavior of mathematics courses and engagement in daily activities and decisions that require arithmetic.

The existence of mathematics anxiety in children is easily assumed but not frequently studied (Jameson, 2013). Both Gierl and Bisanz (1995) and Jameson and Ross (2011) examined mathematics anxiety in early, middle, and upper elementary students and found some self-reported high mathematics anxiety levels. Highly mathematics anxious students had more negative attitudes toward mathematics and lowered mathematics computation scores than students with low mathematics anxiety. Conversely, Krinzinger et al. (2009) found no relationship between mathematics anxiety and mathematics performance in a longitudinal study of children between first and third grade. Evidence is mixed regarding this relationship in elementary school students, and additional research is needed to clarify. Novak and Tassell (2017) also indicated mathematics anxiety as detrimental factor across mathematical domains such as geometry; however, no related study indicated how mathematics anxiety affects students at different levels of geometric thinking.

Theoretical Framework

Many factors affect mathematics achievement, and the related variables that affect geometric achievement and mathematics achievement are the subject of this research. Studies have claimed that mathematics learning strategy factors are significant in improving students' mathematical achievement (Areepattamannil, 2014; Ifamuyiwa & Ajilogba, 2012;

Rabab'h & Veloo, 2014; Sartawi et al., 2012). It is also hypothesized that prior knowledge will directly predict academic performance. Historically, students' performance on standardized tests has been considered an optimal predictor of their subsequent success in school. For example, Pajares (1996) found that previous mathematics attainments were strong predictors of subsequent mathematical performance. Bandura (1997) attributed a very important role to prior knowledge and its effect on students' subsequent attainment. He also emphasized the inclusion of prior knowledge in causal analyses. In recent studies, the effect of students' prior knowledge was not widely considered in the relationship between given psychological variables and mathematical performance (Mousavi et al., 2012). In addition, Chang and Beilock (2016) indicated that mathematics anxiety often resulted in avoidance of mathematics and mathematics-related situations altogether (Ashcraft & Ridley, 2005). Its negative consequences may include: poor performance on standardized mathematics tests and general difficulty with mathematics-related problem-solving (Hambree, 1990); low performance on courses involving numerical reasoning (Núñez-Peña et al., 2013); reduced efficiency in solving simple arithmetic problems (Imbo & Vandierendonck, 2007); or difficulties in basic numerical processing (Maloney, Ansari, et al., 2011; Maloney, Risko, et al., 2010). Further exacerbating these adverse effects is the possibility of a reciprocal relation between mathematics anxiety and mathematics performance as mathematics anxiety interferes with mathematics performance (Ashcraft & Kirk, 2001); poor mathematics performance could in turn increase one's mathematics anxiety (Ma & Xu, 2004). Based on the above, the factors influencing mathematics and geometric achievement are the discussion variables of this research, and it is based on van Hiele's geometry theory.

Van Hiele believed that cognitive progress in geometry could be accelerated by instruction. The progress from one level to the next one is more dependent upon instruction than on age or maturity (Vojkuvkova, 2012). Van Hiele's research is constructing the logical sequence of geometric systems, and his theory is biased toward geometric knowledge content. Van Hiele's model is the basis for many researchers to engage in the development of geometric concepts as the theoretical basis (Clements & Battista, 1992; Clements et al., 1999). Most researchers support the five levels of rationality of the van Hiele model: from intuitive identification to analysis and then advanced to the stage of abstract proof, it can reasonably explain the level of development of children's geometric concepts and can also evaluate students' geometry ability, and the development of geometric concepts is far more affected by teaching than age (Crowley, 1987; Denis, 1987). According to the standard used by Usiskin (1982), when assessing by van Hiele's levels of geometric thinking, students achieve a level of geometric thinking if they correctly answer three-fifths of all items. When a student achieves Level N, but does not achieve Level N+1, then the student is assigned to Level N. If a student passes the test for Level N but does not pass the test for Level N–1, it is a leap that is classified as disqualified.

Methodology

Participants

A total of 108 children of fifth grades (51 boys, 57 girls) from two elementary schools from the central area in Taiwan participated in this study. Of them, 78 children were identified as Level 1 learners (n = 40) or Level 2 learners (n = 38). The other 30 students who lack sufficient cognitive techniques (i.e., not meeting the first level of geometric thinking and not able to solve geometry problems) did not achieve Level 1 in the van Hiele geometric thinking level test and were identified as Level 0; they were not included in the analysis. Both schools were located far from a city center and in areas of average socioeconomic status.

Instruments

Geometry teaching contents

The geometry teaching contents include eight lessons as follows: (a) vertical and parallel, (b) mathematical angle, (c) perimeter, (d) plane geometry, (e) solid geometry, (f) symmetry graphic, (g) scale, (h) bar graph. The teaching content showed various mathematical geometric concepts. Table 1 shows the teaching steps of plane geometry; all eight lessons followed the steps in the table.

Geometry learning achievement test

All children were assessed using a group-administered test of geometry learning achievement, which was developed based on the required geometric ability of a fourth grader according to the Grades 1–9 Curriculum Guidelines. This test uses geometric concepts of reflection, reproduction, and connection to complete the test of geometry learning achievement. Items include multiple-choice and non-multiple-choice questions, which have a scoring standard, and the average score of two scorers was used as the

Table 1: Steps of the Teaching Process of Plane Geometry

Step	Activities
Preparation	• Prepare the learning aids for the course
Groups construction	1. Pretest 2. Guide the students by the animation of unit 3. Review the features of shapes by PowerPoint and ask the questions of the concept they have learned
The decision of preliminary problem	1. Review the definitions of the circle, triangle, and square by PowerPoint 2. Give out the worksheet 3. Introduce the content of the course
Task assignment	1. Do the test from the worksheet 2. Find the angle in the worksheet together in group and drew it by colored strokes
Problem solving	• Inspect and assist students to finish the worksheet
Test Finishing	1. Present and share the result 2. Students appreciate their work and express their opinions 3. Review the construction of circle
Reflection	• Ask the question to review the content of the course
Ending	• Posttest
Conclusion	• Summarize the content

student's score. The time limit for completing four units (each with 3–5 items) was 40 minutes. Children's scores were used for analyses. Examples of items on the scale included the following:

1. Mary introduces her circular cookies to her classmate, Vicky. And Vicky said: "Do you know how to draw a circle?" Please write down your answer to help Mary answer Vicky's question. (Reflection)
2. Jason said a sentence after seeing Mary, Yvonne, and the cookies which Vicky brought. Which one is wrong? (Connection)
   A. Jason said: Cookies are quadrilateral, so there are four sides and four vertices.
   B. Jason said: Both Mary and Yvonne's cookies are round in shape, and the circle has 1 center and 1 radius.
   C. Jason said: The cookies Vicky brought are round, and if you eat three-quarters, the shape of the remaining cookie is fan-shaped.
   D. Jason said: Yvonne's cookie is also round, reminding me of what Mr. Huang said, "the straight line passing through the center of the circle is the diameter of the circle."

3. Based on the geometric shapes shown on the picture, choose the correct answer. (Reproduction)
   A. The circle has an infinite center.
   B. Squares are equal in length and at right angles.
   C. A triangle greater than 90 degrees is called a right triangle.
   D. There is only one pair of parallel sides in a rectangle.

Van Hiele geometric thinking level test

The van Hiele geometric thinking level test was developed by a research project subsidized by the National Science Council, Executive Yuan (T. P. Wu, 2004). Each level includes the basic geometric concepts of triangles, quadrilaterals, and circles. The test has 70 multiple-choice items and divided into three parts. Each item has four options, and students receive one point for each correct answer. There are 25 items in Level 1, 20 items in Level 2, and 25 items in Level 3. This study determines that a student achieves a certain level of geometric thinking if the student correctly answers three-fifths of the items for that level according to the standard used by Usiskin (1982). The student would achieve Level 1 if he answered at least 15 items correctly in part 1, reach Level 2 if he answered at least 12 items correctly in part 2, and achieve Level 3 if he answered at least 15 items correctly in part 3. The reliability of the test is 0.890 ($p < 0.01$); the validity is 0.872. T. P. Wu (2004) explored Level 1 geometric concept in van Hiele's geometric thinking level of elementary school students in Taiwan from grades 1 to 6. The research results point out: (a) because of the apparent difference between a straight line and a curve, it is easy for students to identify; (b) students have difficulty judging the concept of the direction and position of the graph rotation; (c) it is the easiest for students to recognize circles, followed by triangles, and quadrilaterals with unequal sides the most difficult. The results of T. P. Wu's study were based on a large sample of quantitative data and have not explored the reasons affecting the mathematics and geometric achievements of students at different levels of geometric thinking. It remains to be further verified.

Test of prior knowledge in geometry

The prior knowledge geometry test was developed based on the national curriculum schema. Children were required to choose the appropriate answer in the geometric concepts of reflection, reproduction, and connection. The time limit was 40 minutes; 13 items in 4 units were to be completed.

Mathematics learning strategy scale

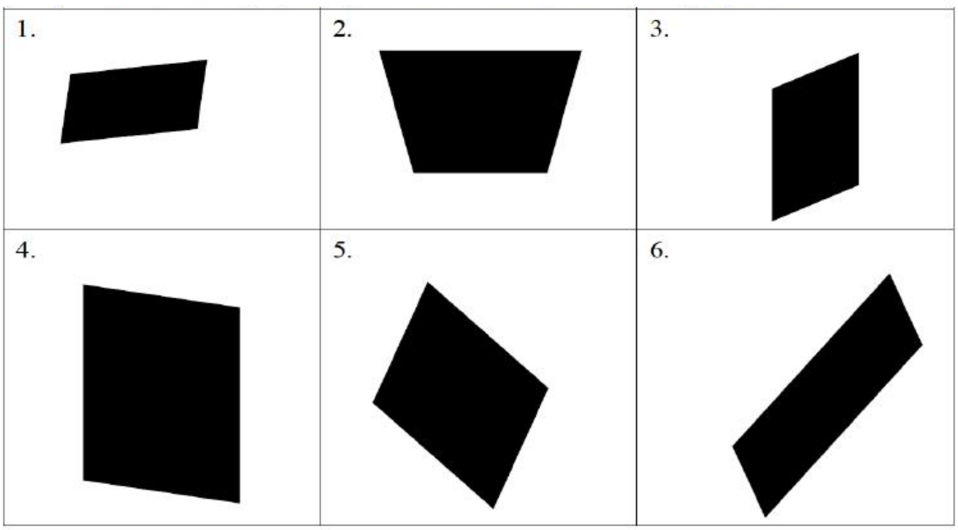
The learning strategy scales (Hopfenbeck & Maul, 2011) were used in the study to identify the learning strategy of students. The internal consistencies of the three strategies (including "elaboration strategies," "memorization strategies" and "control strategies") are .76, .75, and .79 respectively, indicating that the assessment tool has a good internal consistency reliability.

Multiple-choice test of the core concepts of geometry

A nonword-based geometry test of core concepts was administered to measure children's ability to detect intruders in simple pictures by using distance, angle, and sense relations in geometric maps to locate hidden objects. The test used the core concepts mentioned by Dehaene et al. (2006). Forty-five items were included and required a student to choose the picture that violated the concepts of geometry. Since the cross-cultural results of the test showed high consistency at the student's stage of learning, this study used the test for students in central Taiwan. The example is shown in Figure 1.

Figure 1: Example of Multiple-choice Test of the Core Concepts of Geometry

( ) 4. There are 6 graphics. Please find out the different graphics.



Anxiety in Mathematics Scale

All children completed the Children's Anxiety in Mathematics Scale (CAMS) developed by Jameson (2013). The CAMS is a 16-item questionnaire of five-point Likert scale. The questionnaire is further divided into three parts: general mathematics anxiety (GMA), mathematics performance anxiety (MPA), and mathematics error anxiety (MEA). The CAMS was tested on 438 primary school students and analyzed by an exploratory factor analysis using principal components extraction with Varimax rotation; factor loadings of all items were above .3 (.39–.79). Cronbach's alpha was used to evaluate internal consistency, which was high overall (α = .86) and in all three dimensions (α = .73–.81).

Procedure

All children took the test of prior knowledge in geometry at the beginning of the school year to provide an understanding of students' prior knowledge before learning geometry in mathematics classes. A total of 8 lessons in this study were conducted from September 2018 to January 2019. Assessments were conducted at the end of the school year. All students completed the questionnaire and the test of geometry learning achievement within their classroom as posttest. The posttest of geometry learning achievement and core concepts each took 40 minutes; the mathematics learning strategy scale and anxiety in mathematics scale were completed within 40 minutes.

Results

The distribution of scores for each assessment was examined, and the z scores for skewness and kurtosis were calculated. The categories and respective values were as follows: geometry learning achievement ($z = -0.61$ skewness, $z = -0.46$ kurtosis), geometry prior knowledge ($z = -0.21$ skewness, $z = -1.02$ kurtosis), mathematics learning strategy ($z = -0.98$ skewness, $z = 1.26$ kurtosis), core concepts of geometry ($z = -1.48$ skewness, $z = 1.63$ kurtosis) and mathematics anxiety ($z = 0.45$ skewness, $z = 1.02$ kurtosis). Kline (1998) states that a skewness above 3.0 and a kurtosis above 10 indicate serious departures from normality in a distribution. Thus, all distributions of the score assessments were normally distributed.

The group was split based on their achievement in the van Hiele geometric thinking level test into Level 1 learners and Level 2 learners to identify whether these groups differed in mathematics learning strategy, core concepts of geometry, and mathematics anxiety (Table 2).

Table 2: Descriptive Statistics (Mean and Standard Deviations) of Measured Variables for All, Level 1, and Level 2 Learners

	geometry learning achievement	geometry prior knowledge	mathematics learning strategy	core concepts of geometry	mathematics anxiety
All learners (N = 108)	53.40 (18.08)	43.63 (16.84)	3.59 (0.68)	30.34 (8.79)	3.00 (0.95)
Level 1 learners (n = 40)	55.71 (16.82)	38.31 (16.55)	3.85 (0.39)	28.55 (10.50)	3.12 (0.82)
Level 2 learners (n = 38)	58.38 (16.04)	50.58 (16.04)	3.56 (0.58)	34.87 (3.99)	2.67 (0.81)

The groups differed significantly in prior knowledge of geometry with F (1, 76) = 0.84, $p < .01$ and $\eta_p^2 = .13$; mathematics learning strategy with F (1, 76) = 3.45, $p < .05$, and $\eta_p^2 = .08$; core concepts of geometry with F (1, 76) = 17.82, $p < .01$, and $\eta_p^2 = .14$; mathematics anxiety with F (1, 76) = 0.15, $p < .05$, and $\eta_p^2 = .08$. Although Level 2 learners had slightly higher achievement in geometry learning, this difference was not significant as F (1, 76) = 0.84, and $p > .05$. The effect size was calculated using η_p^2, with .01–.06 indicating a small effect, .06–.14 a medium effect, and over .14 a large effect (Cohen, 1988).

Examining Inter-intercorrelations Between Different Abilities

Correlations were determined between various assessments to examine the strength of association between the different abilities. The geometry learning achievement of students was very closely associated with their geometry prior knowledge, mathematics learning strategies, core concepts of geometry, and mathematics anxiety. The core concepts of geometry and mathematics anxiety were slightly more closely associated with their geometry learning achievement than their geometry prior knowledge. Notably, students' mathematics anxiety has a negative correlation with all of these abilities except the mathematics learning strategy (Table 3).

Table 3: Correlations Between Variables for All Learners (*N* = 108)

	1	2	3	4	5
1. geometry learning achievement	–	.52**	.42**	.67**	–.38**
2. geometry prior knowledge		–	.26**	.50**	–.33*
3. mathematics learning strategy			–	.15	–.16
4. core concepts of geometry				–	–.47**
5. mathematics anxiety					–

*p < .05, **p < .01

Further analysis of differences in the geometric thinking of Level 1 and Level 2 learners showed that geometry learning achievement was correlated with geometry prior knowledge, mathematics learning strategy, and core concepts of geometry, regardless of their geometric thinking level. For Level 1 learners, it was positively correlated with prior knowledge and core concepts but negatively with mathematics learning strategy; for Level 2 learners, it was positively correlated with geometry prior knowledge, mathematics learning strategy, and core concepts of geometry. There was a significant difference in the correlation between mathematics anxiety and other abilities of Level 1 learners (Table 4).

Table 4: Correlations for Measured Variables for Level 1 and Level 2 Learners

	1	2	3	4	5
1. geometry learning achievement	–	.61**	–.41**	.89**	–.79**
2. geometry prior knowledge	.77**	–	–.18	.63**	–.56**
3. mathematics learning strategy	.81**	.75**	–	–.46**	.39*
4. core concepts of geometry	.75**	.58**	.60**	–	–.77**
5. mathematics anxiety	–.30	–.18	–.43**	–.12	–

*p < .05, **p < .01

Note: Correlations for Level 2 learners are shown below the diagonal; correlations for Level 1 learners are shown above the diagonal.

The mathematics anxiety of Level 2 learners was not significantly correlated with geometry learning achievement, geometry prior knowledge, and core concepts of geometry and was significantly negatively correlated with mathematics learning strategy. The opposite was found for Level 1 learners: their mathematics anxiety was significantly negatively correlated with their geometry learning achievement, geometry prior knowledge, and core concepts of geometry but was significantly positively correlated with their mathematics learning strategy. This indicates that mathematics anxiety has different effects on learners of different level. In the case of Level 2 learners, whether learners were anxious was not directly correlated with their geometry learning achievement; however, learners that knew how to employ mathematics learning strategy had significantly higher anxiety. In the case of Level 1 learners, learners with lower mathematics anxiety had higher geometry learning achievement. Level 1 learners that knew how to use mathematics learning strategy had significantly lower mathematics anxiety. However, their geometry learning achievement was significantly lower.

Regression Analyses Identifying Differences Between Ability Groups

Hierarchical regression analyses were conducted to examine the variance explained in geometry learning achievement by comparing mathematics learning strategy, core concepts of geometry, and mathematics anxiety, as well as those of different level groups. Overall, mathematics learning strategy and core concepts of geometry contributed significant variance to geometry learning achievement. The explanatory variances of the two variables are .448 and .555, and the *R*-squared changes are .443 and .547, reaching a significant level of .05. The explanatory power of core concepts of geometry for geometric learning achievements is 44.3% and the explanatory power of mathematics learning strategy for geometric learning achievement is 54.7%, while mathematics anxiety explained no variance. In the Level 1 group, the core concepts of geometry and mathematics anxiety significantly contributed to the variance in geometry learning achievement. The explanatory variances of the two levels are .793 and .821, and the *R*-squared changes are .787 and .806, reaching a significant level of .05. The explanatory power of core concepts of geometry for geometric learning achievement is 78.7% and the explanatory power of mathematics anxiety for geometry learning achievement is 80.6%. In the Level 2 group, all abilities significantly contributed to the geometry learning achievement variance except mathematics anxiety. The explanatory variances of the two levels are .565 and .773, and the *R*-squared changes are .553 and .760, reaching a significant level of .05. The explanatory power of core concepts of geometry for geometric learning achievement is 55.3% and the explanatory power of mathematics learning strategy for geometric learning achievement is 76.0% (Table 5).

Furthermore, using prior knowledge in geometry and mathematics anxiety as predictors, the contribution of these variables to the growth in geometry learning achievement was examined (Table 6). Overall, mathematics anxiety explained significant variance in the growth of geometry learning achievement after also explaining prior knowledge in geometry. The explanatory variances of the two levels are .269 and .323, and the *R*-squared changes are .262 and .310, reaching a significant level of .05. The explanatory power of geometry

Table 5: Prediction of Geometry Learning Achievement: Hierarchical Regression Results for All, Level 1, and Level 2 Learners

	R^2	ΔR^2	*p*	Final β
All learners (*N* = 108)				
• core concepts of geometry	.448	.443	.000	.599
• mathematics learning strategy	.555	.547	.000	.326
• mathematics anxiety	.557	.544	.558	−.045
Level 1 learners (*n* = 40)				
• core concepts of geometry	.793	.787	.000	.692
• mathematics learning strategy	.793	.781	.861	.014
• mathematics anxiety	.821	.806	.022	−.266
Level 2 learners (*n* = 38)				
• core concepts of geometry	.565	.553	.000	.416
• mathematics learning strategy	.773	.760	.000	.562
• mathematics anxiety	.773	.753	.921	−.009

Table 6: Predicting Growth in Geometry Learning Achievement Using Geometry Prior Knowledge and Mathematics Anxiety

	R^2	ΔR^2	*p*	Final β
All learners (*N* = 108)				
• geometry prior knowledge	.269	.262	.000	.425
• mathematics anxiety	.323	.310	.005	−.251
Level 1 learners (*n* = 40)				
• geometry prior knowledge	.373	.357	.040	.243
• mathematics anxiety	.671	.653	.000	−.658
Level 2 learners (*n* = 38)				
• geometry prior knowledge	.593	.582	.000	.740
• mathematics anxiety	.619	.597	.135	−.163

prior knowledge for geometry learning achievement is 26.2% and the explanatory power of mathematics anxiety for geometry learning achievement is 31.0%. However, although mathematics anxiety explained significant variance in the growth of geometry learning achievement in the Level 1 group, it did not explain the variance in the Level 2 group. For Level 1, the explanatory variances of the two variables are .373 and .671, and the *R*-squared changes are .357 and .653, reaching a significant level of .05. The explanatory power of geometry prior knowledge for geometry learning achievement is 35.7% and the explanatory power of mathematics anxiety for geometric learning achievement is 65.3%. For Level 2, the explanatory variance of the two variables are .593 and .619, and the *R*-squared changes are .582 and .597. Only geometry prior knowledge reached a significant level of .05, and the explanatory power of geometry prior knowledge for geometry learning achievement is 58.2%.

Discussion

Among all factors related to geometry learning achievement, core concepts of geometry had the highest explanatory power for both Level 1 and Level 2 learners. This finding may very likely be true because knowledge construction of geometry is closely related to its core concepts. Clear and complete core concepts of geometry benefit students' geometry learning achievement. G. J. Hwang et al. (2014) found that using a conceptual map to help students integrate concepts improved their learning achievement. When students' knowledge is constructed from principles and concepts, they can understand and determine how to solve a problem from an integrated perspective when taking the achievement test.

This study found that mathematics anxiety is related to students' low-level geometry learning achievement and is consistent with the results of Núñez-Peña et al. (2013), Wahid et al. (2014), Ramirez et al. (2016), and Lawrence and Williams (2013). Furner and Gonzalez-DeHass (2011) indicated that students' anxiety during mathematics learning could be reduced by teachers' understanding of creating mastery-oriented classrooms. Abdullah and Zakaria (2013) noted that geometry learning achievement is associated with spatial skills, and anxiety is an important factor that affects both variables. Ferguson et al. (2015) stated that spatial anxiety, general anxiety, and small-scale spatial skills are the most robust of the tested predictors of mathematics anxiety. Maloney, Waechter, et al. (2012) further indicated that mathematics anxiety greatly affects spatial processing ability. Therefore, reducing students' mathematics anxiety can be achieved by reducing their spatial and general anxieties and by helping them improve their small-scale spatial skills, which will further improve their geometry learning achievement. Mathematics anxiety can be regarded as an important factor related to Level 1 learners' geometry learning achievement to advance to the next level.

Students with lower levels of geometric thinking have higher mathematics anxiety, which may be caused by their unfamiliar or incomprehension of the geometric concepts; it may even affect mathematics enjoyment, motivation, and self-confidence (Núñez-Peña et al., 2013). A further recommendation is that if instructors want to reduce learners' mathematics anxiety, there are several strategies mentioned by Blazer (2011) for students (using good study techniques and focusing on past successes), teachers (relating mathematics to real life and accommodating students' varied learning styles) and parents (focusing on past successes and demonstrating positive uses for mathematics).

After further analysis of factors related to students' geometry learning achievement with different levels of geometric thinking, mathematics anxiety has a high negative explanatory power for the geometry learning achievement of students with low-level geometric thinking. In contrast, prior knowledge in geometry and mathematics learning strategy had high explanatory power for the geometry learning achievement of Level 2 learners. Previous studies already noted that mathematics anxiety influences students' learning achievement in mathematics (Núñez-Peña et al., 2013; Wahid et al., 2014) and that anxiety further influences students' working memory (Eysenck et al., 2007; Smeding et al., 2015). Mathematics learning strategy can be regarded as an important factor related to Level 2 learners' geometry learning achievement to advance to the next level.

This study found that learning strategy is also an important factor related to learning achievement in mathematics, which is consistent with the findings of Lin and Tai (2015), Cleary and Chen (2009), and Muelas and Navarro (2015). This finding shows that Level 2 learners choose a suitable mathematics learning strategy to improve their geometry performance. Compared with Level 1 learners, Level 2 learners understand the meaning and have the mastery of the question. The important factor that affects Level 2 learners to advance to the next level is learning strategy. The above literature points out the importance of learning strategies for mathematics learning. C. Lau et al. (2015) noted that students with high learning achievement had more independent learning processes in mathematics and a greater ability for self-regulation compared with students with low learning achievement. Therefore, teachers can help students develop more independent learning strategies by using control strategies, such as setting goals and thinking in advance, to find key points of mathematics problems and to solve them. For the suggestions on learning strategies, Behzadi et al. (2014) mentioned that successful mathematics learning strategies (e.g., PQ4R Strategy or MURDER Strategy) are learning strategies that teachers could apply in the course.

Furthermore, in the Level 1 group, mathematics anxiety significantly negatively correlated with geometry learning achievement. This finding shows that students with higher anxiety when encountering mathematics concepts had poorer geometry learning achievement. In contrast, if students' mathematics anxiety could be alleviated, it would effectively improve Level 1 learners' geometry learning achievement. Many studies have shown the effects of mathematics anxiety on geometry learning achievement (Núñez-Peña et al., 2013). This study further found that when the core concepts of geometry were considered, mathematics anxiety still negatively correlated with Level 1 learners' geometry learning achievement. Hence, lowering students' mathematics anxiety is important for improving their geometry learning achievement. In addition, students' sense of self-efficacy had an enormous effect on their anxiety management (Galla & Wood, 2012). If providing assistance can help students improve their self-efficacy, it will also improve their low learning achievement caused by mathematics anxiety. Tests are important to students with different ability levels, particularly for dividing students into different levels. However, test scores may not accurately reflect a learner's ability (Logan et al., 2011). The influence of mathematics anxiety on students' geometry learning achievement may be greater.

Conclusion

Understanding the influence of factors that affect geometry learning achievement in students with different levels of geometric thinking and providing suitable interventions to help them achieve their learning goals will improve their learning performance. The findings of this study indicate that improving the mathematics learning environment to lower students' mathematics anxiety may be beneficial to learners with low-level geometric thinking and may further improve their geometry learning achievement. Teachers should understand the learning characteristics of Level 1 learners, create a friendly mathematics learning environment, and utilize teaching methods that are most appropriate for students' cognitive development. These will help learners to garner a better geometry learning achievement. For students with high levels of geometric thinking, prior knowledge in geometry and mathematics learning strategy have a significant effect on their geometry learning achievement. Hence, before teaching geometric concepts, teachers should verify that learners have prior knowledge of the lesson and help learners develop and use mathematics learning strategy that fits their learning style. This process will enable high-level learners to apply learning strategies, construct a complete geometric knowledge system, and further gain higher geometry learning achievement.

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數學焦慮和學習策略是影響國小學生幾何學習的關鍵因素

黃思華

摘 要

本研究探討幾何核心概念、學習策略、數學焦慮如何預測幾何學習成就，以及各變項能否解釋不同幾何思考層次學生的學習成就。研究共有 108 名學生（10–11 歲）參與，他們完成 van Hiele 幾何思考層次、幾何先備知識、幾何學習成就等測驗，以及數學學習策略、幾何核心概念和數學焦慮量表。根據 van Hiele 幾何思考層次測驗，共有 78 名學生分別屬幾何思考層次第一級（$n = 40$）和第二級（$n = 38$）。本研究共進行八堂課，每堂課的內容為不同的幾何單元。研究結果發現，數學焦慮和數學學習策略分別是影響幾何思考層次第一級和第二級學生幾何學習成就的關鍵因素，結果突顯了影響幾何思考層次第一級和第二級學習者幾何學習成就的不同因素，以及數學焦慮和數學學習策略影響學習者幾何學習成就的重要性。

關鍵詞：幾何核心概念；學習成效；學習策略；數學焦慮；van Hiele 幾何思考層次

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